

CHAPTER 25

ELECTRIC CHARGE AND COULOMB'S LAW

W

e begin here a detailed study of electromagnetism, which will extend throughout most of the remainder of this text. Electromagnetic forces are responsible for the structure of atoms and for the binding of atoms in molecules and solids. Many properties of materials that we have studied so far are electromagnetic in their nature, such as the elasticity of solids and the surface tension of liquids. The spring force, friction, and the normal force all originate with the electromagnetic force between atoms.

Among the examples of electromagnetism that we shall study are the force between electric charges, such as occurs between an electron and the nucleus in an atom; the motion of a charged body subject to an external electric force, such as an electron in an oscilloscope beam; the flow of electric charges through circuits and the behavior of circuit elements; the force between permanent magnets and the properties of magnetic materials; and electromagnetic radiation, which ultimately leads to the study of optics, the nature and propagation of light.

In this chapter we begin with a discussion of electric charge, some properties of charged bodies, and the fundamental electric force between two charged bodies.

25-1 ELECTROMAGNETISM: A PREVIEW

What do the following have in common?

1. You turn on the light switch in your room. The consumption of fuel at a power plant produces electromagnetic energy by causing a loop of electrically conducting wire to rotate in the vicinity of a magnet. Ultimately some of this energy is transferred to the electrons in the filament of your lightbulb, which can transform the electrical energy into visible light.

2. You enter a command on your computer keyboard. A stream of electrons is formed to transmit your instructions. There are many thousands of possible pathways for the electrons through the computer circuitry, but most are blocked by electronic gates. Electrons can move only through the gates that have been opened by your command

so that the stream of electrons reaches its destination and your command is executed.

3. You push the channel select button on the remote control of your TV set. Electromagnetic waves travel from the remote control unit to a receptor on the set, which then tunes the set to accept another electromagnetic wave that originates from a satellite orbiting high above the Earth. The waves from the satellite provide instructions for your set to use electric and magnetic forces to focus and direct a beam of electrons that strikes the surface of the picture tube and produces a visible image.

The common factor in these diverse phenomena is that they all depend on forces that we describe as *electric* or *magnetic* to control and direct the flow of energy or particles. These forces form the basis of our study of *electromagnetism*. We will find in our study that all electromagnetic effects can be explained by a set of four basic

equations, called *Maxwell's equations*. These equations represent individual laws of electromagnetism, just as we have previously discussed equations that represent Newton's laws of mechanics or the laws of thermodynamics.

Our study will first consider electric phenomena and then magnetic phenomena. Later we will show that the two cannot be separated; certain electric phenomena produce magnetic effects, and certain magnetic phenomena produce electric effects. This leads us to unify electric and magnetic phenomena under the common name of electromagnetism. The development of the laws of electromagnetism and their unification was a great triumph of 19th-century physics. Their application has led directly to a great range of devices of practical use, such as motors, radios and televisions, radar, microwave ovens, and cellular phones.

The development of electromagnetic theory continued in the 20th century with three very significant advancements. In 1905, Albert Einstein showed that, to a moving observer, electric effects could appear as magnetic effects, and thus observers in relative motion could disagree in assigning their measurements to electric or magnetic causes. This conclusion formed the basis of the special theory of relativity, which ultimately was to revolutionize our concepts of space and time. The second development was the introduction of a quantum theory of electromagnetism, called *quantum electrodynamics*, which reached its fruition around 1949 and enabled properties of the atom to be calculated with incredible precision, currently about 11 significant figures. The third development of the 20th century was the unification of electromagnetism with another force, called the "weak" force, which is responsible for certain ra-

dioactive decay processes and other interactions between particles. Just as electric and magnetic effects were unified into the electromagnetic interaction, so electromagnetic and weak effects were shown in the 1960s to be unified under the *electroweak* interaction. For our study of electric and magnetic forces, however, the electroweak interaction does not yield anything new, and it is more convenient to consider the separate electromagnetic interaction.

Figure 25-1 is a time line of some of the major events in the development of our understanding of electromagnetism.

25-2 ELECTRIC CHARGE

After you pass a plastic comb through your hair a few times, you will find that the comb can exert a force on individual strands of your hair. You may also observe that, once the strands of hair are attracted to the comb and come into contact with it, they may no longer be attracted to it.

It seems reasonable to conclude that the attraction between the comb and the hair is a result of some physical entity being transferred from one to the other when they rub together, with the same physical entity being transferred back again to neutralize the attraction when they come into contact. This physical entity is called *electric charge*, and today we understand this transfer on the basis of electrons that can be removed from the atoms of one object and attached to the atoms of the other object.

The transfer of electric charge by means of friction is a commonly observed phenomenon. It was known to the ancient Greeks, who observed that pieces of amber rubbed with fur could attract bits of straw. When you walk across a carpet and are shocked by touching a metal door knob, or when a lightning flash stretches between a cloud and the ground, you are observing the effects of this transfer of charge.

When we "charge" an object (that is, when we transfer charge to it), we find that it can exert a force on another charged object. Early observations that this force can be either attractive or repulsive led to the conclusion that there are two kinds of electrical charge, which are called positive and negative.*

Although effects resulting from the transfer of charge can be powerful, it is remarkable that they originate from the transfer of only a tiny fraction of the electric charge that is contained in objects. Ordinary matter is made of electrically neutral atoms or molecules that contain equal amounts of positive charge (the nucleus) and negative charge (the electrons). When two objects rub together, relatively few electrons from the atoms of one object are

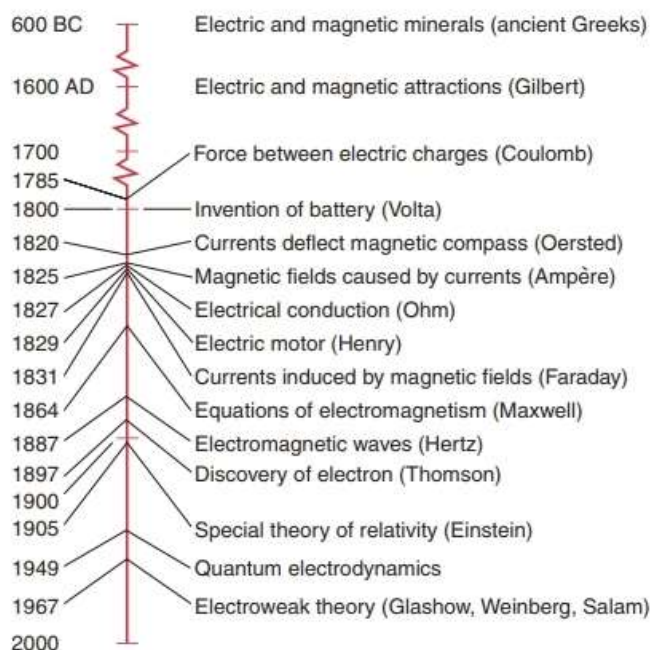


FIGURE 25-1. Time line of major developments in electromagnetism.

* The positive and negative labels for electric charge were chosen arbitrarily by Benjamin Franklin (1706–1790) who, among his other accomplishments, was a scientist of international reputation. In fact, his scientific reputation may have enabled his diplomatic triumphs in France during the American War of Independence.

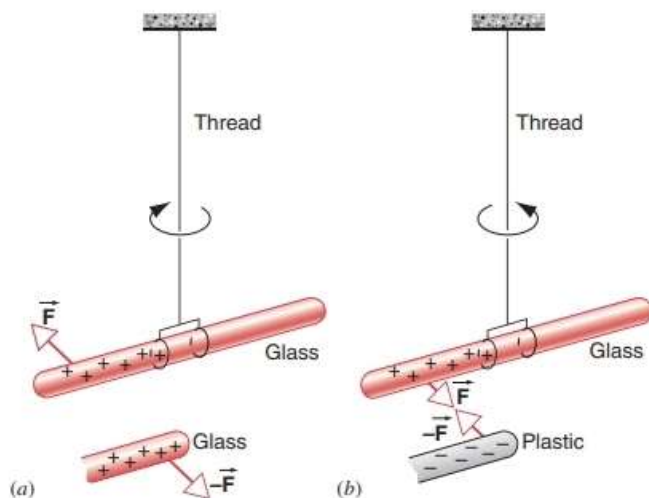


FIGURE 25-2. (a) Two similarly charged rods repel each other. (b) Two oppositely charged rods attract each other.

transferred to the other; most of the electrons remain undisturbed. It is this slight upset of the balance between the enormous but equal amounts of positive and negative charge in an object that is responsible for most commonly observed electrical effects.

When we rub a plastic rod with fur, electrons are transferred to the rod; because it has an excess of electrons (which carry a negative charge), the rod becomes negatively charged. The fur now has a deficiency of electrons and so it is positively charged. We can see the attraction of the rod for individual strands of the fur, which results from the charge on each. In a similar way, we can rub a glass rod with silk and observe that both become charged and can attract one another. In each case, we have transferred a relatively small number of electrons and upset the electrical neutrality of these objects.

Let us charge a glass rod by rubbing one end of it with silk and then suspend it from a thread, as in Fig. 25-2. If we place a similarly charged glass rod nearby, we find that the two rods repel one another, as in Fig. 25-2a. However, if we place a charged plastic rod (charged by rubbing with fur) nearby, the two rods attract one another, as in Fig. 25-2b.

We account for the existence of these two kinds of forces in terms of two kinds of charge. When plastic is rubbed with fur, electrons are transferred to the plastic and it becomes negatively charged. When glass is rubbed with silk, electrons are transferred to the silk, leaving the glass with a deficiency of electrons and therefore a net positive charge. The forces observed in Fig. 25-2 can be summarized by the following rule:

Charges of the same sign repel one another, and charges of the opposite sign attract one another.

In Section 25-4, we put this rule into quantitative form, as Coulomb's law of force. We consider only charges that are either at rest with respect to each other or moving very

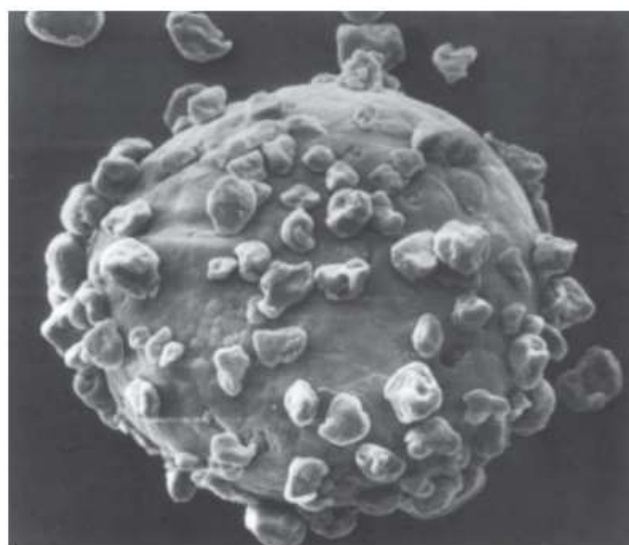


FIGURE 25-3. A carrier bead from a Xerox photocopier, covered with toner particles that stick to it by electrostatic attraction. The diameter of the bead is about 0.3 mm.

slowly, a restriction that defines the subject of *electrostatics*.

Electrical forces between charged bodies have many industrial applications, including electrostatic paint spraying and powder coating, fly-ash precipitation, nonimpact ink-jet printing, and photocopying. Figure 25-3 for example, shows a tiny carrier bead in a photocopying machine, covered with particles of black powder called *toner*, that stick to the carrier bead by electrostatic forces. These negatively charged toner particles are eventually attracted from their carrier beads to a positively charged latent image of the document to be copied, which is formed on a rotating drum. A charged sheet of paper then attracts the toner particles from the drum to itself, after which they are heat-fused in place to make the final copy.

The net electric charge of an object is usually represented by the symbol q . The charge is a scalar quantity. It can be positive or negative, depending on whether the object has a net positive or negative charge. Electric charge is measured in units of coulombs (C). The coulomb is a very large unit of charge; it takes about 6×10^{18} electrons to make one coulomb of charge.

The coulomb cannot be derived from previously defined units. Because electric charge is a new quantity, we are free to define its basic unit in any convenient way. One possible way would be in terms of the force exerted between two standard charges at a given separation, such as the quantity of charge that exerts a force of one newton on a similar charge a distance of one meter away. However, the force between static charges is difficult to measure, and so in practice it is more useful to define the coulomb in terms of the magnetic force between current-carrying wires (which is discussed in Chapter 33). This force can be measured more precisely than the electric force between static

charges. It is therefore more convenient to define an SI base unit in terms of current (rate of flow of electric charge per unit time). The coulomb as a unit for electric charge is then a derived unit, obtained from the fundamental units of current and time (see Appendix A).

Electric Charge Is Quantized

When we transfer electric charge from one object to another, the transfer cannot be done in arbitrarily small units. That is, the flow of charge as a current is not a continuous flow, but is made up of discrete elements.* Experiments show that the electric charge always exists only in quantities that are integer multiples of a certain elementary quantity of charge e . That is,

$$q = ne \quad n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (25-1)$$

where (to four significant figures)

$$e = 1.602 \times 10^{-19} \text{ C}.$$

The *elementary charge* e is one of the fundamental constants of nature whose experimental value has been determined to an uncertainty of about 4 parts in 10^8 .

The electron and the proton are examples of commonly occurring particles that each carry one fundamental unit of charge. The electron has a charge of $-e$ and the proton has a charge of $+e$. Some particles, such as the neutron, carry no net electric charge. Other elementary particles are known that carry charges that are small multiples of e , usually ± 1 , ± 2 , or ± 3 . Each particle has a corresponding *antiparticle*, which has the same mass but the opposite electric charge; the antielectron, which is known as the *positron*, has a charge of $+e$. Antiparticles do not commonly exist in nature, but can be created in decays and reactions of nuclei and elementary particles.

Equation 25-1 tells us that it is possible to have a net charge on an object of $+10e$ or $-6e$ but never $3.57e$. When the values of a property are restricted to discrete multiples of a basic quantity, we say that the property is *quantized*.

Because the elementary charge is small, under ordinary circumstances we are not aware of the discrete nature of the flow of charge. For example, in a wire of an electronic circuit in which small currents of one milliampere are typical, 6×10^{15} electrons pass through any cross section of the wire every second!

Ordinary atoms are electrically neutral, which means that they contain equal quantities of positive and negative charge. The nucleus of the atom contains Z protons (where Z is called the *atomic number* of the atom) and thus a charge of $+Ze$. In a neutral atom, Z negatively charged

electrons circulate about the nucleus. It is often possible to remove one or more electrons from an atom, creating an ion that has an excess positive charge of $+e$, $+2e$, $\dots$. For example, if we could remove all of the electrons from an atom of uranium ($Z = 92$), we would create a particle with a charge of $+92e$. Under certain circumstances, we can even attach an extra electron to a neutral atom, creating a negatively charged ion.

Although we believe electrons to be fundamental particles with no substructure, protons are not fundamental particles. They are made up of more elementary entities called *quarks*. The quarks are assigned fractional electric charges of $-\frac{1}{3}e$ and $+\frac{2}{3}e$. The proton is composed of three quarks, two with charges of $+\frac{2}{3}e$ and one with charge of $-\frac{1}{3}e$, which add up to a net charge of $+e$. Experimental evidence for the existence of quarks inside the proton is very strong (for example, high-energy electrons can be made to scatter from the fractionally charged quarks inside the proton), but, no matter how violently the protons are made to collide, no free quark has been released. As a result, no free particle with a fractional charge has ever been observed. This fact can be understood if the attractive force that one quark exerts on another *increases* with their separation. This is in contrast to the electromagnetic and gravitational forces, both of which *decrease* as the distance between a pair of interacting bodies increases.

SAMPLE PROBLEM 25-1. A penny, being electrically neutral, contains equal amounts of positive and negative charge. What is the magnitude of these equal charges?

Solution The charge q is given by NZe , in which N is the number of atoms in a penny and Ze is the magnitude of the positive and the negative charges carried by each atom.

The number N of atoms in a penny, assumed for simplicity to be made of copper, is $N_A m/M$, in which N_A is the Avogadro constant. The mass m of the coin is 3.11 g, and the mass M of 1 mol of copper (called its *molar mass*) is 63.5 g. We find

$$\begin{aligned} N &= \frac{N_A m}{M} = \frac{(6.02 \times 10^{23} \text{ atoms/mol})(3.11 \text{ g})}{63.5 \text{ g/mol}} \\ &= 2.95 \times 10^{22} \text{ atoms.} \end{aligned}$$

Every neutral atom has a negative charge of magnitude Ze associated with its electrons and a positive charge of the same magnitude associated with its nucleus. Here e is the elementary charge, $1.60 \times 10^{-19} \text{ C}$, and Z is the atomic number of the element in question. For copper, Z is 29. The magnitude of the total negative or positive charge in a penny is then

$$\begin{aligned} q &= NZe = (2.95 \times 10^{22})(29)(1.60 \times 10^{-19} \text{ C}) \\ &= 1.37 \times 10^5 \text{ C.} \end{aligned}$$

This is an enormous charge. By comparison, the charge that you might get by rubbing a plastic rod is perhaps 10^{-9} C , smaller by a factor of about 10^{14} . For another comparison, it would take 1–2 days for a charge of $1.37 \times 10^5 \text{ C}$ to flow through the filament of a typical lightbulb. There is a lot of electric charge in ordinary matter.

* In Franklin's day, electric charge was thought to be a substance and to flow as a continuous fluid. Today we know that fluids are made up of individual atoms and molecules—matter is discrete. Similarly, the "electric fluid" is not continuous but discrete.

25-3 CONDUCTORS AND INSULATORS

Materials are commonly classified based on the ability of electrons to flow through them. In some materials, such as metals, electrons can move relatively freely. We call these materials *conductors*. Electrons deposited at one location in the material can easily move throughout the material. Other examples of conductors include tap water and the human body.

In other materials, electrons can hardly flow at all. Electrons deposited at one location will remain at that location. We call these materials *insulators*. Examples of insulators include glass, plastics, and many crystalline materials such as NaCl.

If you try to charge a copper rod by holding it in your hand and rubbing it with fur, you will not be successful. Electrons may be transferred from the rod to the fur as a result of friction, but additional electrons will easily flow from your body through your hand into the rod to replace those that were removed. As a result, no net charge builds up on the rod due to rubbing. We can consider the Earth to possess an infinite supply of electrons, some of which can flow into your body to replace those that have been lost to the rod. When we have a pathway through which electrons can flow between an object and the Earth, the object is said to be electrically *grounded*.

If instead we attach a plastic handle to the copper rod, we find that we can build up a charge by rubbing it. The insulating handle blocks the flow of electrons between the rod and your body.

Isolated atoms of a conducting material such as copper generally contain loosely bound electrons that can easily be detached, leaving a positively charged ion. When cop-

per atoms come together to form solid copper, these loosely bound electrons do not remain attached to individual atoms but become free to wander through the material. These free electrons are called *conduction electrons*; in copper, a typical conductor, there are about 10^{23} conduction electrons per cm^3 . The positively charged ions are not free to move and remain fixed within the solid lattice structure of the copper.

The experiment of Fig. 25-4 demonstrates the mobility of charge in a conductor. An uncharged copper rod is suspended by an insulating thread. When a positively charged glass rod is brought near one end of the copper, the mobile conduction electrons in the copper are attracted by the positive charges on the glass. The flow of electrons to the near end of the copper leaves the far end with a deficiency of electrons and a net positive charge. The negatively charged end of the copper rod and the positively charged glass exert attractive forces on one another. Note that this situation is very different from that of Fig. 25-2; in Fig. 25-4, the glass attracts a copper rod that carries no net charge. (As we will discuss in the next section, the electrical force depends inversely on the separation between the charges; therefore the attractive force between the glass and the negative end of the copper rod is much stronger than the repulsive force between the glass and the positive end of the copper rod.)

If instead of a positively charged glass rod in Fig. 25-4 we used a negatively charged plastic rod, the effect would be the same: an attractive force between the plastic and the copper. In this case the negatively charged plastic would repel the conduction electrons of the copper, leaving the near end of the copper with a positive charge. There would be an attractive force between the negatively charged plastic and the positively charged end of the copper rod.

It is also possible to have an attractive force between a charged body and an uncharged insulator. Figure 25-5a shows a charged comb attracting uncharged bits of paper. The explanation for this attraction is different from that of the attraction between the glass rod and the copper rod. In this case, the paper is an insulator, and it is not possible for electrons to collect on one end of the paper (as was the case for the conductor in Fig. 25-4). Instead, electrons in individual molecules in the bits of paper are repelled by the negatively charged comb, and so the electrons are preferentially located on the side of each molecule that is away from the comb. In each molecule, the positive end (the end with a lack of electrons) is closest to the comb and feels a greater force of attraction to the comb. This is responsible for the net attractive force between the comb and the paper (Fig. 25-5b). The same attractive force would occur if the comb were positively charged.

The separation of positive and negative charge in an isolated object under the influence of a nearby charged object is known as *polarization*. Polarization can occur at the macroscopic level, such as in the copper rod of Fig. 25-4, or at the molecular level, as in Fig. 25-5.

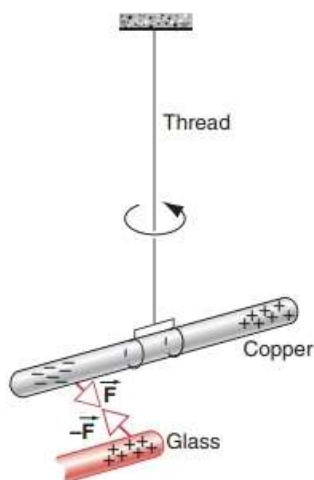


FIGURE 25-4. Either end of an isolated uncharged copper rod is attracted by a charged rod of either sign. In this case, conduction electrons in the copper rod are attracted to the near end of the copper rod, leaving the far end with a net positive charge.

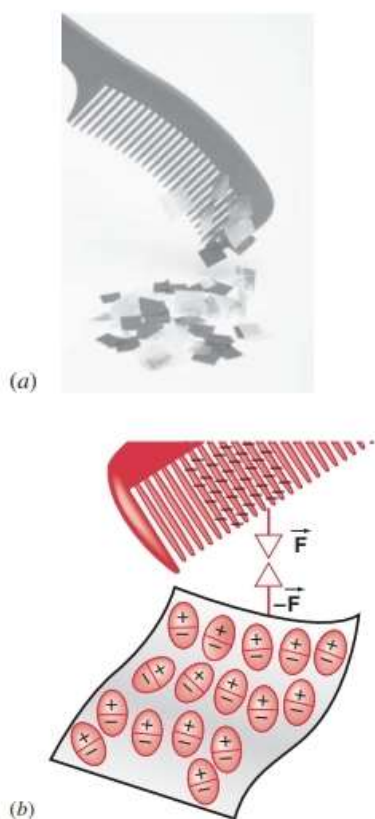


FIGURE 25-5. (a) A charged comb attracts uncharged bits of paper. (b) The negatively charged comb polarizes the charges in the molecules, resulting in an attractive force between the comb and the paper.

Charging by Contact and by Induction

Suppose now we touch a positively charged glass rod to an uncharged copper rod (as in Fig. 25-6). Electrons will flow from the copper to neutralize the positive charges on the glass. However, because electrons do not flow through the glass, they can neutralize only those positive charges at the point of contact with the copper. To remove additional electrons from the copper, we can “wipe” the glass rod along the copper rod, thereby transferring electrons to fresh, unneutralized areas of the glass that come into contact with the copper (Fig. 25-6a). If we then remove the glass rod, the copper remains with a deficiency of electrons and therefore a net positive charge. Electrons will flow through the copper so that the positive charges (the ion cores) are evenly distributed along the surface of the copper. This direct transfer of charge from one object to another is called *charging by contact*. Even though negative electrons were actually transferred, it is often convenient to regard the experiment shown in Fig. 25-6 as if positive charges were transferred from the glass rod to the copper.

Let us return to the situation of Fig. 25-4. If we attach a wire between the positive end of the copper and ground (as

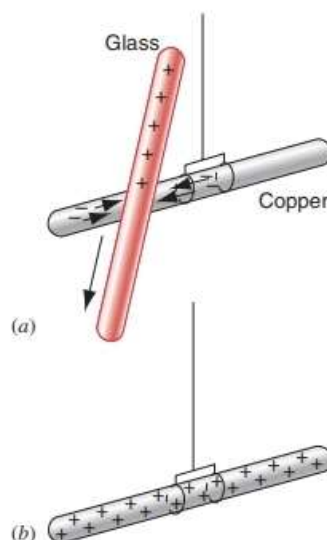


FIGURE 25-6. (a) Charging by contact. Electrons flow from the copper to neutralize positive charges at the point of contact with the glass. (b) The resulting charge on the copper when the glass is removed.

in Fig. 25-7a), electrons will flow from ground to neutralize the positive charges in the copper. Leaving the glass rod in place, if we then remove the connection to ground, the copper rod retains a net negative charge. If we then remove the glass rod the negative charges will distribute themselves over the surface of the copper (Fig. 25-7b) to be as far apart from one another as possible. This method of charging an object is called *charging by induction*. Note that we have been able to use the positively charged glass rod to transfer either positive charge to the copper by contact or negative charge (from ground) by induction.

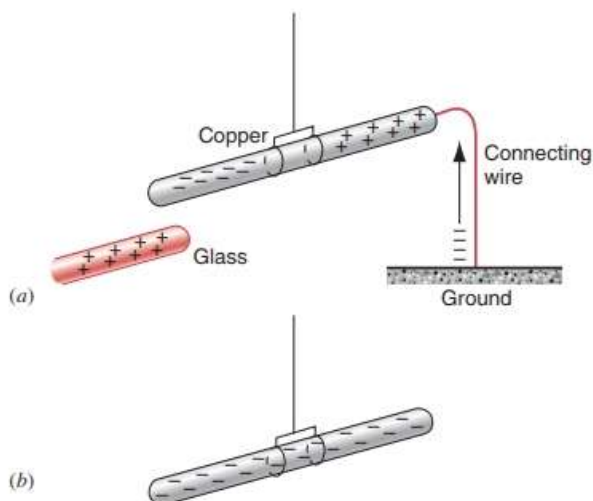


FIGURE 25-7. (a) Charging by induction. Electrons flow from ground to neutralize the positive charges on the far end of the copper rod. (b) The resulting charge on the copper when the glass is removed.

25-4 COULOMB'S LAW

So far in this chapter we have established that there are two kinds of electric charge and that charges exert forces on each other. It is now our goal to understand the nature of this force.

The first successful quantitative experiments to study the force between electric charges were done by Charles Augustin Coulomb (1736–1806), who measured electrical attractions and repulsions and deduced the law that governs them. In principle, Coulomb's apparatus is similar to Fig. 25-2, except that he used small charged spheres indicated by a and b in Fig. 25-8.

If a and b are charged, the electric force on a tends to twist the suspension fiber. Coulomb cancelled out this twisting effect by turning the suspension head through the angle θ needed to keep the two charges at a particular separation. The angle θ is then a relative measure of the electric force acting on charge a . The device of Fig. 25-8 is a *torsion balance*; a similar arrangement was used later by Cavendish to measure gravitational attractions (Section 14-3).

Experiments by Coulomb and his contemporaries showed that the electrical force exerted by one charged body on another depends directly on the product of the magnitudes of the two charges and inversely on the square of their separation. That is,

$$F \propto \frac{|q_1||q_2|}{r^2}.$$

Here F is the magnitude of the mutual force that acts on each of the two charges q_1 and q_2 , and r is the distance be-

tween their centers. The force on each charge due to the other acts along the line connecting the charges. As required by Newton's third law, the force exerted by q_1 on q_2 is equal in magnitude but opposite in direction to the force exerted by q_2 on q_1 , even though the magnitudes of the charges may be different.

To turn the above proportionality into an equation, we introduce a constant of proportionality K , which we can call the Coulomb constant. We thus obtain, for the force between the charges,

$$F = K \frac{|q_1||q_2|}{r^2}. \quad (25-2)$$

Equation 25-2, which is called *Coulomb's law*, generally holds only for charged objects whose sizes are much smaller than the distance between them. We often say that it holds only for *point charges*.

Our belief in Coulomb's law does not rest quantitatively on Coulomb's experiments. Such measurements could not, for example, convince us that the exponent of r in Eq. 25-2 is exactly 2 and not, say 2.0001. In Section 27-7 we show that Coulomb's law can also be deduced from an indirect experiment, which shows that, if the exponent in Eq. 25-2 is not exactly 2, it differs from 2 by at most 1×10^{-16} .

Coulomb's law resembles Newton's inverse square law of gravitation, $F = Gm_1m_2/r^2$, which was already 100 years old at the time of Coulomb's experiments. Both are inverse square laws, and the charge q plays the same role in Coulomb's law that the mass m plays in Newton's law of gravitation. One difference between the two laws is that gravitational forces are always attractive, whereas electrostatic forces can be repulsive or attractive, depending on whether the two charges have the same or opposite signs.

In the SI system, the constant K is expressed in the following form:

$$K = \frac{1}{4\pi\epsilon_0}. \quad (25-3)$$

Although the choice of this form for the constant K appears to make Coulomb's law needlessly complex, it ultimately results in a simplification of formulas of electromagnetism that are used more often than Coulomb's law.

The constant ϵ_0 , which is called the *electric constant* (also known as the *permittivity*), has a value that is determined by the value of the speed of light, as we discuss in Chapter 39. Its exact value is

$$\epsilon_0 = 8.85418781762 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.$$

The Coulomb constant K has the corresponding value (to three significant figures)

$$K = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2.$$

With this choice of the constant K , Coulomb's law can be written

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}. \quad (25-4)$$

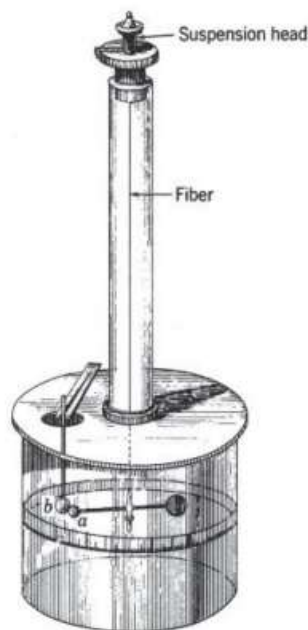


FIGURE 25-8. Coulomb's torsion balance, from his 1785 memoir to the Paris Academy of Sciences.

When K has the above value, expressing q in coulombs and r in meters gives the force in newtons.

The significance of Coulomb's law goes far beyond the description of the forces exerted by charged spheres on each other. This law, when incorporated into the structure of quantum physics, correctly describes (1) the electrical forces that bind the electrons of an atom to its nucleus, (2) the forces that bind atoms together to form molecules, and (3) the forces that bind atoms and molecules together to form solids or liquids. Thus most of the forces of our daily experience that are not gravitational in nature are electrical.

SAMPLE PROBLEM 25-2. In Sample Problem 25-1 we saw that a copper penny contains both positive and negative charges, each of a magnitude $1.37 \times 10^5 \text{ C}$. Suppose that these charges could be concentrated into two separate bundles, held 100 m apart. What attractive force would act on each bundle?

Solution From Eq. 25-4 we have

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q|^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.37 \times 10^5 \text{ C})^2}{(100 \text{ m})^2} = 1.69 \times 10^{16} \text{ N}.$$

This is about 2×10^{12} tons of force! Even if the charges were separated by one Earth diameter, the attractive force would still be about 120 tons. In all of this, we have sidestepped the problem of forming each of the separated charges into a "bundle" whose dimensions are small compared to their separation. Such bundles, if they could ever be formed, would be blasted apart by mutual Coulomb repulsion forces.

The lesson of this sample problem is that you cannot disturb the electrical neutrality of ordinary matter very much. If you try to pull out any sizable fraction of the charge contained in a body, a large Coulomb force appears automatically, tending to pull it back.

SAMPLE PROBLEM 25-3. The average distance r between the electron and the proton in the hydrogen atom is $5.3 \times 10^{-11} \text{ m}$. (a) What is the magnitude of the average electrostatic force that acts between these two particles? (b) What is the magnitude of the average gravitational force that acts between these particles?

Solution (a) From Eq. 25-4 we have, for the electrostatic force,

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(5.3 \times 10^{-11} \text{ m})^2} = 8.2 \times 10^{-8} \text{ N}.$$

Although this force may seem small (it is about equal to the weight of a speck of dust), it produces an enormous acceleration of the electron within the atom, about 10^{23} m/s^2 .

(b) For the gravitational force, we have

$$\begin{aligned} F_g &= G \frac{m_e m_p}{r^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(5.3 \times 10^{-11} \text{ m})^2} \\ &= 3.6 \times 10^{-47} \text{ N}. \end{aligned}$$

We see that the gravitational force is weaker than the electrostatic force by a factor of about 10^{39} . Although the gravitational force is weak, it is always attractive. Thus it can act to build up very large masses, as in the formation of stars and galaxies, so that large gravitational forces can develop. The electrostatic force, on the other hand, is repulsive for charges of the same sign, so that it is not possible to accumulate large concentrations of either positive or negative charge. We must always have the two types of charge together, so that they largely compensate for each other. The charges that we are accustomed to in our daily experiences are slight disturbances of this over-riding balance.

SAMPLE PROBLEM 25-4. The nucleus of an iron atom has a radius of about $4 \times 10^{-15} \text{ m}$ and contains 26 protons. What repulsive electrostatic force acts between two protons in such a nucleus if they are separated by a distance of one radius?

Solution From Eq. 25-4 we have

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(4 \times 10^{-15} \text{ m})^2} = 14 \text{ N}.$$

The large repulsive electrostatic force, more than 3 lb and acting on a single proton, must be balanced by the attractive nuclear force that binds the nucleus together. This force, whose range is so short that its effects cannot be felt very far outside the nucleus, is known as the "strong nuclear force" and is very well named.

Coulomb's Law: Vector Form

So far we have considered only the magnitude of the force exerted by one charge on another, determined according to Coulomb's law. Force, being a vector, has directional properties as well. In the case of Coulomb's law, the direction of the force is determined by the relative sign of the two electrical charges.

As illustrated in Fig. 25-9, suppose we have two point charges q_1 and q_2 separated by a distance r_{12} . For the moment, we assume the two charges to have the same sign, so that they repel one another. Let us consider the force *on* particle 1 exerted *by* particle 2, which we write in our usual form as $\vec{F}_{12}$. The position vector that locates particle 1 relative to particle 2 is $\vec{r}_{12}$; that is, if we were to define the origin of our coordinate system at the location of particle 2, then $\vec{r}_{12}$ would be the position vector of particle 1.

If the two charges have the same sign, then the force is repulsive and, as shown in Fig. 25-9a, $\vec{F}_{12}$ must be parallel to $\vec{r}_{12}$. If the charges have opposite signs, as in Fig. 25-9b, then the force $\vec{F}_{12}$ is attractive and antiparallel to $\vec{r}_{12}$. In either case, we can represent the force as

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}. \quad (25-5)$$

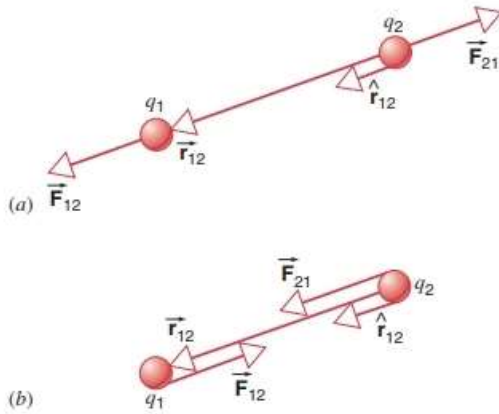


FIGURE 25-9. (a) Two point charges q_1 and q_2 of the same sign exert equal and opposite repulsive forces on one another. The vector $\vec{F}_{12}$ locates q_1 relative to q_2 , and the unit vector $\hat{r}_{12}$ points in the direction of $\vec{F}_{12}$. Note that $\vec{F}_{12}$ is parallel to $\vec{r}_{12}$. (b) The two charges now have opposite signs, and the force is attractive. Note that $\vec{F}_{12}$ is antiparallel to $\vec{r}_{12}$.

Here r_{12} represents the magnitude of the vector $\vec{F}_{12}$, and $\hat{r}_{12}$ indicates the *unit vector* in the direction of $\vec{F}_{12}$. That is,

$$\hat{r}_{12} = \frac{\vec{F}_{12}}{r_{12}}. \quad (25-6)$$

We used a form similar to Eq. 25-5 to express the gravitational force (see Eqs. 14-2 and 14-3).

One other feature is apparent from Fig. 25-9. According to Newton's third law, the force exerted *on* particle 2 *by* particle 1, $\vec{F}_{21}$, is opposite to $\vec{F}_{12}$. This force can then be expressed in exactly the same form:

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}. \quad (25-7)$$

Here $\hat{r}_{21}$ is a unit vector that points from particle 1 to particle 2; that is, it would be the unit vector in the direction of particle 2 if the origin of coordinates were at the location of particle 1.

The vector form of Coulomb's law is useful because it carries within it the directional information about $\vec{F}$ and whether the force is attractive or repulsive. Using the vector form is of critical importance when we consider the forces acting on an assembly of more than two charges. In this case, Eq. 25-5 would hold for every pair of charges, and the total force on any one charge would be found by taking the *vector sum* of the forces due to each of the other charges. For example, the force on particle 1 in an assembly would be

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots, \quad (25-8)$$

where $\vec{F}_{12}$ is the force on particle 1 from particle 2, $\vec{F}_{13}$ is the force on particle 1 from particle 3, and so on. Equation 25-8 is the mathematical representation of the *principle of superposition* applied to electric forces. It asserts that the force acting on one charge due to another is independent of whether or not other charges are present, and therefore we

can calculate the force separately for each pair of charges and then take their vector sum to find the net force on any charge. For instance, the force $\vec{F}_{13}$ that particle 3 exerts on particle 1 is completely unaffected by the presence of particle 2. The principle of superposition is not at all obvious and may fail in the case of very strong electric forces. Only through experiment can its applicability be verified. For all situations we meet in this text, however, the principle of superposition is valid.

SAMPLE PROBLEM 25-5. Figure 25-10 shows three charged particles, held in place by forces not shown. What electrostatic force, due to the other two charges, acts on q_1 ? Take $q_1 = -1.2 \mu\text{C}$, $q_2 = +3.7 \mu\text{C}$, $q_3 = -2.3 \mu\text{C}$, $r_{12} = 15 \text{ cm}$, $r_{13} = 10 \text{ cm}$, and $\theta = 32^\circ$.

Solution This problem calls for the use of the superposition principle. We start by computing the magnitudes of the forces that q_2 and q_3 exert on q_1 . We substitute the magnitudes of the charges into Eq. 25-5:

$$\begin{aligned} F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r_{12}^2} \\ &= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.2 \times 10^{-6} \text{ C})(3.7 \times 10^{-6} \text{ C})}{(0.15 \text{ m})^2} \\ &= 1.77 \text{ N}. \end{aligned}$$

The charges q_1 and q_2 have opposite signs so that the force exerted by q_2 on q_1 is attractive. Hence $\vec{F}_{12}$ points to the right in Fig. 25-10.

We also have

$$\begin{aligned} F_{13} &= \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.2 \times 10^{-6} \text{ C})(2.3 \times 10^{-6} \text{ C})}{(0.10 \text{ m})^2} \\ &= 2.48 \text{ N}. \end{aligned}$$

These two charges have the same (negative) sign so that the force exerted by q_3 on q_1 is repulsive. Thus $\vec{F}_{13}$ points as shown in Fig. 25-10.

The components of the resultant force $\vec{F}_1$ acting on q_1 are determined by the corresponding components of Eq. 25-8, or

$$\begin{aligned} F_{1x} &= F_{12x} + F_{13x} = F_{12} + F_{13} \sin \theta \\ &= 1.77 \text{ N} + (2.48 \text{ N})(\sin 32^\circ) = 3.08 \text{ N} \end{aligned}$$

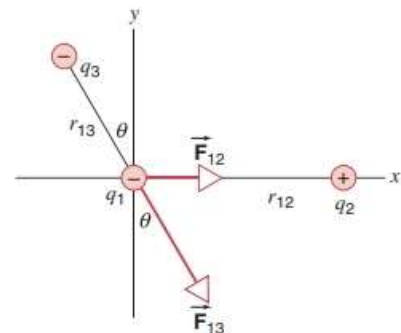


FIGURE 25-10. Sample Problem 25-5. The three charges exert three pairs of action–reaction forces on each other. Only the two forces acting on q_1 are shown here.

and

$$\begin{aligned} F_{1y} &= F_{12y} + F_{13y} = 0 - F_{13} \cos \theta \\ &= -(2.48 \text{ N})(\cos 32^\circ) = -2.10 \text{ N}. \end{aligned}$$

From these components, you can show that the magnitude of $\vec{F}_1$ is 3.73 N and that this vector makes an angle of -34° with the x axis.

25-5 CONTINUOUS CHARGE DISTRIBUTIONS

So far we have seen how to calculate the forces due to point charges. In many applications, however, electric forces are exerted by charged objects in the form of rods, plates, or solids. For simplicity we assume that the objects are insulators and that charge is spread throughout the surface or volume of the object, forming a *continuous charge distribution*.

Figure 25-2 showed forces exerted by one charged rod on another. Coulomb's law applies only to point charges, and we therefore cannot use Coulomb's law in its point charge form to calculate the force exerted by one charged rod on the other. It is possible to imagine the rods to be covered with point charges and to use Coulomb's law to calculate the force exerted by each point charge of one rod on each point charge of the other, but such an approach would be hopelessly complicated—if the rods carry the small charge of only 1 nC, it would be necessary to consider 10^{10} point charges on each rod!

Instead, we return to an idea from Franklin's time and regard the charge as a continuous property. The basic procedure is to divide the charge into infinitesimal elements and use the methods of calculus to find the total force due to all the elements.

If an object contains a net charge q , we imagine it to be divided into many small elements dq . Each element has a certain length, area, or volume, depending on whether we are considering charges that are respectively distributed in one, two, or three dimensions. We express dq in terms of the size of the element and the *charge density*, which describes how the charges are distributed over the length, area, or volume of the object. For most of the problems we consider in this text, the charges will be distributed uniformly over the object, which means that the charge density has the same value everywhere on the object.

In some situations, charges are distributed in one dimension, such as the thin charged rods of Fig. 25-2. In this case, we express dq in terms of the *linear charge density* (charge per unit length) λ , whose basic unit is C/m. A small element of the rod of length dx has a charge dq given by

$$dq = \lambda dx. \quad (25-9)$$

If the rod is uniformly charged, so that a total charge q is spread evenly over its length L , then $\lambda = q/L$. For example,

a rod of length $L = 0.12 \text{ m}$ carrying a uniformly distributed charge of $q = 5.4 \times 10^{-6} \text{ C}$ has a linear charge density of $\lambda = q/L = 4.5 \times 10^{-5} \text{ C/m}$. A short piece of the rod of length $dx = 1.0 \text{ mm}$ would then carry a charge $dq = \lambda dx = 4.5 \times 10^{-8} \text{ C}$.

In other situations, charge might be distributed over a two-dimensional area, such as the surface of the carrier bead in Fig. 25-3. In this case, dq is expressed in terms of the *surface charge density* (charge per unit area) σ , measured in the SI unit of C/m^2 . A small element of area dA would then have a charge given by

$$dq = \sigma dA. \quad (25-10)$$

If a charge q is spread uniformly over a surface of area A , then $\sigma = q/A$.

The charge also might be spread throughout the volume of a three-dimensional object. In this case, we use the *volume charge density* (charge per unit volume) ρ , whose SI unit is C/m^3 . The charge dq in an element of volume dV would then be

$$dq = \rho dV. \quad (25-11)$$

If the charge q is distributed uniformly throughout the volume V , then $\rho = q/V$.

To illustrate these concepts, we will calculate expressions for the force exerted by a continuous charge distribution on a point charge q_0 . By extending these methods, it is possible to calculate the force exerted by one continuous charge distribution on another.

The procedure for finding the force exerted by a continuous charge distribution on a point charge is as follows:

1. Imagine the continuous charge distribution to be divided into a large number of small charge elements.
2. Pick an arbitrary charge element and express its charge dq in terms of Eq. 25-9, 25-10, or 25-11, depending respectively on whether the charge is distributed over a line, an area, or a volume.
3. Because dq is infinitesimally small, we can treat it as a point charge. Express the magnitude of the force element dF exerted by the charge dq on the charge q_0 in terms of Coulomb's law, Eq. 25-4:

$$dF = \frac{1}{4\pi\epsilon_0} \frac{|dq||q_0|}{r^2}, \quad (25-12)$$

where r is the distance between dq and q_0 .

4. By taking into account the signs and locations of dq and q_0 , determine the direction of the force element $d\vec{F}$.

5. The total force is then found by adding all the infinitesimal force elements, which involves the integral

$$\vec{F} = \int d\vec{F}. \quad (25-13)$$

In performing this integral, we usually need to take into account that different charge elements dq may give force elements $d\vec{F}$ in different directions. Equation 25-13 really

means three different equations for the three components of $\vec{F}$:

$$F_x = \int dF_x, \quad F_y = \int dF_y, \quad F_z = \int dF_z. \quad (25-14)$$

Occasionally we can use arguments based on symmetry to avoid calculating one or two of these integrals.

A Uniform Line of Charge

Figure 25-11 shows a thin rod of length L that lies along the z axis and carries a uniformly distributed positive charge q , so that its linear charge density is $\lambda = q/L$. We want to find the force exerted by the rod on a positive point charge q_0 located on the perpendicular bisector of the rod (the positive y axis) a distance y from its center.

The figure shows the results of performing steps 1, 2, and 3 of our procedure. We imagine the rod divided into small elements of length dz . An arbitrary element of charge $dq = \lambda dz$ is located a distance z from its center and exerts a force dF on q_0 , where

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r^2}.$$

The direction of the force $d\vec{F}$ is shown in the figure. There is no component of $d\vec{F}$ in the x direction (perpendicular to the page), so $F_x = 0$. We can also use a symmetry argument to show that $F_z = 0$. For every charge element dq located at position $+z$, there is another charge element dq located at $-z$. When we add the forces due to the charge elements at $+z$ and $-z$, we find that the z components have equal magnitudes but point in opposite directions, so their sum is zero. Because the charge q_0 is located in the median plane of the rod, this cancellation will occur for every such pair of charge elements along the entire length of the rod. We therefore conclude that $F_z = 0$.

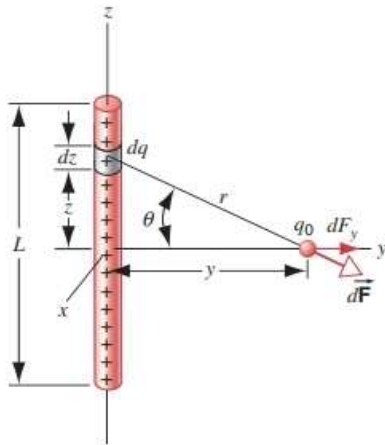


FIGURE 25-11. A uniformly charged rod. To find the force on the point charge q_0 , we consider the rod to consist of many individual charge elements such as dq .

Only F_y remains to be calculated. The element $dF_y = dF \cos \theta$ is shown in Fig. 25-11. With $dq = \lambda dz$, $r^2 = y^2 + z^2$, and $\cos \theta = y/r$, we have

$$dF_y = dF \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda dz}{(y^2 + z^2)} \frac{y}{\sqrt{y^2 + z^2}}$$

$$F_y = \int dF_y = \frac{1}{4\pi\epsilon_0} q_0 \lambda y \int_{-L/2}^{L/2} \frac{dz}{(y^2 + z^2)^{3/2}}.$$

Evaluating the integral (see Appendix I and note that y is a constant), we obtain

$$F_y = \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{y\sqrt{y^2 + L^2/4}}. \quad (25-15)$$

This force is in the positive y direction when q_0 and q are positive. If the charge q_0 is moved to other locations in the xy plane, the expression for the force may change (see Exercise 14).

It is often instructive to evaluate expressions such as this one in various limiting cases. Let us consider the result when $y \gg L$, in which case the force becomes

$$F_y \approx \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{y^2},$$

which is just the expression for the force of one point charge on another. When we are very far from the charged rod, or when the rod is very small, it looks like a point charge.

A Ring of Charge

Figure 25-12 shows a thin ring of radius R carrying a uniformly distributed positive charge q , so that its linear charge density is $\lambda = q/2\pi R$. We wish to find the force exerted by the ring on a positive point charge q_0 located on the axis of the ring (which we choose as the positive z axis) a distance z from the center of the ring. A small charge

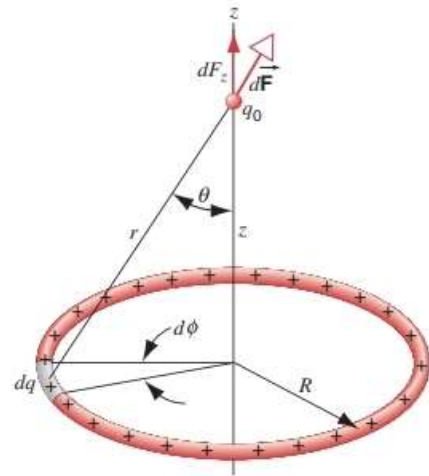


FIGURE 25-12. A uniformly charged ring. To find the force on a point charge q_0 , we consider the ring to consist of many individual charge elements such as dq .

element of the ring has a length $R d\phi$ and thus carries a charge $dq = \lambda R d\phi$. The force dF exerted on q_0 by dq is

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q_0 dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R d\phi}{(z^2 + R^2)}.$$

We can use a symmetry argument to establish that the only nonzero component of $\vec{F}$ is its z component. For every element dq of the ring, there will be another element of equal charge dq at the opposite end of a diameter through the center of the ring; when the force elements on q_0 due to these two charge elements are added, all force components other than F_z will cancel. With $\cos \theta = z/r$, we find

$$\begin{aligned} F_z &= \int dF_z = \int dF \cos \theta \\ &= \int \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R d\phi}{(z^2 + R^2)} \frac{z}{\sqrt{z^2 + R^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_0 \lambda R z}{(z^2 + R^2)^{3/2}} \int_0^{2\pi} d\phi. \end{aligned}$$

The integral evaluated around the ring gives 2π , and so the final result for the force is

$$F_z = \frac{1}{4\pi\epsilon_0} \frac{q_0 q z}{(z^2 + R^2)^{3/2}}. \quad (25-16)$$

Is this result also valid if q_0 is located on the negative z axis? (See Exercise 15.)

We can examine this result in the limiting case as $z \rightarrow \infty$. For $z \gg R$, we obtain

$$F_z \approx \frac{1}{4\pi\epsilon_0} \frac{q_0 q}{z^2},$$

which again gives the result for point charges. When we are very far from the ring, it appears to be a point charge.

Note also that $F_z = 0$ for $z = 0$. This is reasonable, because at the center of the ring the charge q_0 would be pushed equally in all directions by the charge elements that make up the ring.

A Disk of Charge

Figure 25-13 shows a circular disk of radius R that carries a positive charge q uniformly distributed over its surface, so that its surface charge density is $\sigma = q/\pi R^2$. A positive point charge q_0 is located on the axis of the disk (the positive z axis) a distance z from the center of the disk. To find the force exerted by the disk on the point charge, we can divide the disk into a series of concentric rings. The charge on the ring of radius w and thickness dw shown in Fig. 25-13 is $dq = \sigma dA = \sigma(2\pi w dw) = 2\pi\sigma w dw$. The force dF_z on q_0 exerted by this ring can be found using Eq. 25-16 with q replaced by dq and R replaced with w :

$$dF_z = \frac{1}{4\pi\epsilon_0} \frac{q_0(2\pi\sigma w dw)z}{(z^2 + w^2)^{3/2}}.$$

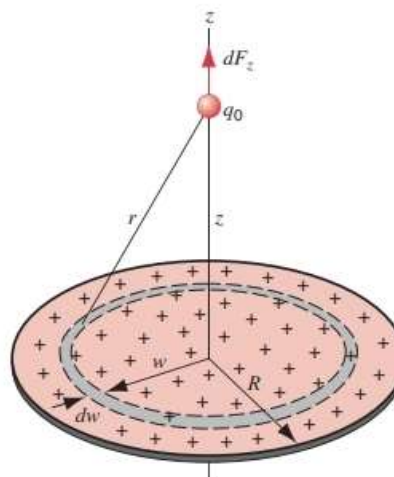


FIGURE 25-13. A circular disk carrying a uniform surface charge density. The force on a point charge q_0 is determined by dividing the disk into thin circular rings.

To add the force elements due to all of the rings, we integrate as w ranges from 0 to R :

$$\begin{aligned} F_z &= \frac{1}{4\pi\epsilon_0} q_0 2\pi\sigma z \int_0^R \frac{w dw}{(z^2 + w^2)^{3/2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{2q_0 q}{R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right). \end{aligned} \quad (25-17)$$

Note that the integral is of the form $\int u^{-3/2} du$, which can be evaluated directly. How would this equation differ for $z < 0$? (See Exercise 15.) As $z \rightarrow \infty$, you can use the binomial expansion (see Appendix I) to show that this expression reduces to Coulomb's law for point charges.

In these three examples, we have assumed that all charges are positive. If either the point charge or the extended object (but not both!) carry negative charge, the direction of the force is opposite to what is shown in Figs. 25-11 to 25-13.

SAMPLE PROBLEM 25-6. Two circular disks of radius $R = 5.0$ cm are separated by 6.0 cm along a common vertical axis. The disks carry equal but opposite electric charges distributed uniformly over their surfaces. How much charge q must be placed on each disk to suspend a tiny drop of oil of mass 4.0×10^{-15} kg and charge $-e$ at a point along the axis of the disks and midway between them?

Solution Let the top disk be positively charged and the bottom disk be negatively charged. Then the top disk attracts the drop and the bottom disk repels it. Because the drop is midway between the disks, the forces have the same magnitude F_z given by Eq. 25-17. To hold the drop in equilibrium, the net upward electrostatic force $2F_z$ must equal the weight mg of the drop. Setting these two forces equal and using Eq. 25-17, we obtain

$$q = \frac{mg}{\frac{4e}{4\pi\epsilon_0 R^2} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)}.$$

Solving, we find $q = 35 \text{ nC}$.

This method was used by Robert A. Millikan in a series of experiments begun in 1906 to measure the charge of the electron. (See Section 26-6.)

A Special Case

There is one special case in which a continuous charge distribution can be treated as a point charge, which allows us to use Coulomb's law in its point-charge form. This occurs when the charge is distributed with spherical symmetry. That is, the volume charge density may vary with radius, but the density is uniform in a thin shell at any radius.

First we consider a thin spherical shell. In Section 14-5 we established two properties of the gravitational force exerted by a uniformly dense spherical shell of matter on a point mass. These properties are: (1) the force on a particle inside the shell is zero, and (2) the force on an external particle is the same as if the entire mass of the shell were concentrated at its center.

The symmetry between the gravitational and electrostatic force laws (both depend on $1/r^2$) will allow us to make many analogies between gravitation and electrostatics. We can often apply results from gravitation directly to electrostatics without additional calculation or proof. This is true for the properties of uniform shells. The proofs of these two important results for electrostatics follow exactly the corresponding proofs in Section 14-5 for the gravitational force.

A uniformly charged spherical shell exerts no electrostatic force on a point charge located anywhere inside the shell.

A uniformly charged spherical shell exerts an electrostatic force on a point charge outside the shell as if all the charge of the shell were concentrated in a point charge at its center.

There is one difference between the gravitational and electrostatic cases: the gravitational force is always attractive, but the electrostatic force may be either attractive or repulsive. However, this difference does not affect the transfer of the above two rules from the gravitational to the electrostatic force.

These rules can be used to obtain a related result that is valid for spherical charge distributions. Suppose we have a spherical distribution of charge in which the volume charge density ρ is either constant or varies only as a function of the radius r . We can therefore regard the sphere as being composed of a set of thin spherical shells. Each shell is uniformly charged; that is, the charge density of one shell may

differ from the charge density of another shell, but on each individual shell the charge is uniformly distributed. Then we can apply each of the two rules to every shell of the sphere. If our test charge is somewhere inside the sphere, the shells outside of the charge exert no force on it according to the first rule. (This result was used for the gravitational force in Sample Problem 14-4.) If the test charge is outside the sphere, then every shell can be replaced by a point charge at the center, and so the entire sphere can be replaced by a point charge equal to the total charge of the sphere.

For this reason the force exerted by the nucleus of an atom on its electrons generally cannot give us any information about the distribution of positive charge within the nucleus. For spherical nuclei in which the charge density depends only on r , all charge distributions give identical forces on an electron outside the nucleus. However, occasionally an electron may wander *inside* the nucleus and give us information about its distribution of positive charge.

SAMPLE PROBLEM 25-7. The spherical nucleus of a certain atom contains a positive charge Ze in a volume of radius R . Compare the force exerted on an electron inside the nucleus at radius $0.5R$ with the force at radius R for a nucleus in which (a) the charge density is constant throughout its volume, and (b) the charge density increases in direct proportion to the radius r .

Solution (a) For the uniformly charged nucleus (which is a good approximation to the behavior of many nuclei), the volume charge density is

$$\rho = \frac{q}{V} = \frac{Ze}{\frac{4}{3}\pi R^3}.$$

When the electron is at $r = R$, the entire nucleus can be replaced by a point charge $q = Ze$ located at its center, so the force $F(r)$ at $r = R$ is

$$F(R) = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{R^2}.$$

When the electron is at $r = R/2$, the charge at all larger radii exerts no force on the electron (by the first rule for spherical shells). The charge inside $R/2$ can be replaced with a point charge. How much charge is inside $R/2$? Since the volume of a sphere depends on r^3 , the volume inside $r = R/2$ is $(1/2)^3 = 1/8$ the volume of the sphere. So the charge inside $r = R/2$ is $1/8$ the charge of the entire sphere, and we conclude that

$$\frac{F(R/2)}{F(R)} = \frac{1}{8}.$$

(b) We can write the charge density as $\rho(r) = br$. First we must evaluate the constant of proportionality b . We know that the total charge on the nucleus must be Ze , so

$$\int_0^R \rho dV = \int_0^R (br) 4\pi r^2 dr = Ze,$$

where $dV = 4\pi r^2 dr$ is the volume of a spherical shell. Carrying out the integral, we find $b = Ze/\pi R^4$.

From the second rule for spherical shells, we know that the force $F(R)$ is the same for both charge distributions. However,

$F(R/2)$ will be different for the two distributions. To find $F(R/2)$, we need to know how much charge q' is contained within the sphere of radius $R/2$, since the charge outside of radius $R/2$ exerts no force on the electron. This charge is

$$q' = \int_0^{R/2} \rho dV = \int_0^{R/2} \frac{Ze r}{\pi R^4} 4\pi r^2 dr = \frac{Ze}{16}.$$

The force on the electron at $r = R/2$ can be found by replacing the sphere within $R/2$ by a point charge q' at its center, which gives a force $1/16$ the force at the surface:

$$\frac{F(R/2)}{F(R)} = \frac{1}{16}.$$

This result is very different from that of the uniform sphere in part (a), demonstrating that, although the electron outside the nucleus cannot distinguish between the two distributions, the electron inside clearly can.

Atomic electrons can occasionally penetrate the nucleus, and electrons can be fired into the nucleus by an accelerator. These two methods can give us information about the distribution of charge within the nucleus. One result of these experiments is that the charge density is found to be very nearly uniform for most nuclei. Despite the Coulomb repulsion of the protons (which is expected to drive them toward the nuclear surface) and despite the strong nuclear force between the protons (which is expected to make them congregate near the center of the nucleus), the protons in the nucleus are distributed with a roughly uniform density. Furthermore, this density is approximately the same for light nuclei as it is for heavy nuclei. These surprising results offer insight into important properties of the nuclear force.

25-6 CONSERVATION OF CHARGE

When a glass rod is rubbed with silk, a positive charge appears on the rod. Measurement shows that a corresponding negative charge appears on the silk. This suggests that rubbing does not create charge but merely transfers it from one object to another, disturbing slightly the electrical neutrality of each. This hypothesis of the *conservation of charge* has stood up under careful experimental tests both for large-scale objects and for atoms, nuclei, and elementary particles. No exceptions have ever been found.

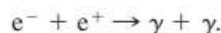
In analogy with other conservation laws, such as conservation of momentum or conservation of energy, we can express conservation of electric charge as

$$\sum q = \text{constant} \quad \text{or} \quad q_i = q_f. \quad (25-18)$$

In any process occurring in an isolated system the net initial charge must equal the net final charge. In finding the net charge, it is important to take into account the signs of the individual charges.

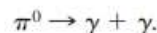
An interesting example of charge conservation comes about when an electron (charge = $-e$) and an antielectron

or positron (charge = $+e$) are brought close to each other. The two particles may annihilate one another, converting all their rest energy into radiant energy. The radiant energy may appear in the form of two gamma rays (high-energy packets of electromagnetic radiation, which are chargeless):



The net charge is zero both before and after the event, and charge is conserved.

Certain uncharged particles, such as the neutral π meson, sometimes decay into two gamma rays:

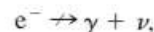


This decay conserves charge, the total charge again being 0 before and after the decay. For another example, a neutron ($q = 0$) decays into a proton ($q = +e$) and an electron ($q = -e$) plus another neutral particle, a neutrino ($q = 0$):



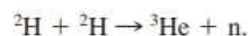
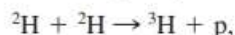
The total charge is zero, both before and after the decay, and charge is conserved. Experiments have been done to search for decays of the neutron into a proton with no electron emitted, which would violate charge conservation. No such events have been found.

The decay of an electron ($q = -e$) into neutral particles, such as gamma rays (γ) or neutrinos (ν), is forbidden; for example



because that decay would violate charge conservation. Attempts to observe this decay have likewise been unsuccessful, indicating that, if the decay does occur, the electron must have a lifetime of at least 10^{23} years!

Another example of charge conservation is found in the fusion of two deuterium nuclei ${}^2\text{H}$ (called “heavy hydrogen”) to make helium. Among the possible reactions are



The deuterium nucleus contains one proton and one neutron and therefore has a charge of $+e$. The nucleus of the isotope of hydrogen with mass 3, written ${}^3\text{H}$ and known as *tritium*, contains one proton and two neutrons, and thus also has a charge of $+e$. The first reaction therefore has a net charge of $+2e$ on each side and conserves charge. In the second reaction, the neutron is uncharged, while the nucleus of the isotope of helium with mass 3 contains two protons and one neutron and therefore has a charge of $+2e$. The second reaction thus also conserves charge. Conservation of charge explains why we never see a proton emitted along with ${}^3\text{He}$ or a neutron along with ${}^3\text{H}$.

To sum up, charge is conserved in *all* known interactions among particles; no exceptions have ever been observed.

MULTIPLE CHOICE

25-1 Electromagnetism: A Preview

25-2 Electric Charge

- Electric charges A and B are attracted to each other. Electric charges B and C repel each other. If A and C are held close together they will
 - attract.
 - repel.
 - not affect each other.
 - More information is needed to answer.
- Electric charges A and B are attracted to each other. Electric charges B and C are also attracted to each other. If A and C are held close together they will
 - attract.
 - repel.
 - not affect each other.
 - More information is needed to answer.
- Electric charges A and B repel each other. Electric charges B and C also repel each other. If A and C are held close together they will
 - attract.
 - repel.
 - not affect each other.
 - More information is needed to answer.

25-3 Conductors and Insulators

- If an object made of substance A rubs an object made of substance B , then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C , then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C ?
 - B becomes positively charged and C becomes positively charged.
 - B becomes positively charged and C becomes negatively charged.
 - B becomes negatively charged and C becomes positively charged.
 - B becomes negatively charged and C becomes negatively charged.
- A positively charged rod is held near a ball suspended by an insulating thread. The ball is seen to swing toward the charged rod. What can be concluded?
 - The ball must have had a charge opposite to that of the rod.
 - The ball may have been neutral originally, but it became charged when the rod was held near it.
 - The ball must be a conductor.
 - The ball is not positively charged, but it could be neutral.
- A spherical conducting ball is suspended by a grounded conducting thread. A positive point charge is moved near the ball. The ball will
 - be attracted to the point charge and swing toward it.
 - be repelled from the point charge and swing away from it.
 - not be affected by the point charge.
- A spherical conducting ball is suspended by an insulating thread. A positive point charge is moved near the ball. The ball will
 - be attracted to the point charge and swing toward it.

- be repelled from the point charge and swing away from it.
- not be affected by the point charge.

25-4 Coulomb's Law

- A $3\text{-}\mu\text{C}$ point charge q_1 is located a distance d away from a $-6\text{-}\mu\text{C}$ point charge q_2 . What is the ratio $|\vec{F}_{12}|/|\vec{F}_{21}|$?
 - $1/2$
 - 1
 - 2
 - 18
- Two 200-pound lead balls are separated by a distance of 1 m. Both balls have the same positive charge q . What charge will produce an electrostatic force between the balls that is of the same order of magnitude as the weight of one ball?
 - $1 \times 10^{-14} \text{ C}$
 - $1 \times 10^{-7} \text{ C}$
 - $3 \times 10^{-4} \text{ C}$
 - $2 \times 10^{-2} \text{ C}$
- Two identical, small, conducting spheres are separated by a distance of 1 m. The spheres originally have the same positive charge, and the force between them is F_0 . Half of the charge on one sphere is then moved to the other sphere. The force between the spheres is now
 - $F_0/4$.
 - $F_0/2$.
 - $3F_0/4$.
 - $3F_0/2$.
 - $3F_0$.
- Two identical, small, conducting spheres are separated by a distance of 1 m. The spheres originally have equal but opposite charges, and the force between them is F_0 . Half of the charge on one sphere is then moved to the other sphere. The force between the spheres is now
 - $F_0/4$.
 - $F_0/2$.
 - $3F_0/4$.
 - $3F_0/2$.
 - $3F_0$.

25-5 Continuous Charge Distributions

- A point charge q is located a distance a from the surface of a sphere of radius $2a$. A charge Q is distributed uniformly throughout the volume of the sphere. The magnitude of the electrostatic force between the point charge q and the sphere is F , where
 - $F = |qQ|/4\pi\epsilon_0 a^2$.
 - $|qQ|/4\pi\epsilon_0 a^2 > F > |qQ|/12\pi\epsilon_0 a^2$.
 - $|qQ|/12\pi\epsilon_0 a^2 > F > |qQ|/20\pi\epsilon_0 a^2$.
 - $|qQ|/20\pi\epsilon_0 a^2 > F > |qQ|/36\pi\epsilon_0 a^2$.
 - $F = |qQ|/36\pi\epsilon_0 a^2$.

25-6 Conservation of Charge

- A positively charged rod is held near a neutral conducting sphere suspended by an insulating thread. The sphere will
 - be unaffected, because it is neutral.
 - remain neutral, but be repelled from the rod anyway.
 - remain neutral, but be attracted to the rod anyway.
 - acquire a negative charge and be repelled from the rod.
 - acquire a negative charge and be attracted to the rod.
- Objects A , B , and C are three identical, insulated, spherical conductors. Originally A and B both have charges of $+3 \text{ mC}$, while C has a charge of -6 mC . Objects A and C are allowed to touch, then they are moved apart. Then objects B and C are allowed to touch, and they are moved apart.
 - If objects A and B are now held near each other, they will
 - attract.
 - repel.
 - have no effect on each other.
 - If instead objects A and C are held near each other, they will
 - attract.
 - repel.
 - have no effect on each other.

QUESTIONS

1. You are given two metal spheres mounted on portable insulating supports. Find a way to give them equal and opposite charges. You may use a glass rod rubbed with silk but may not touch it to the spheres. Do the spheres have to be of equal size for your method to work?
2. In Question 1, find a way to give the spheres equal charges of the same sign. Again, do the spheres need to be of equal size for your method to work?
3. A charged rod attracts bits of dry cork dust, which, after touching the rod, often jump violently away from it. Explain.
4. How would your answers to Multiple-choice question 1, Multiple-choice question 2, and Multiple-choice question 3 change if any of the objects *A*, *B*, or *C* could be uncharged?
5. The experiments described in Section 25-2 could be explained by postulating four kinds of charge—that is, on glass, silk, plastic, and fur. What is the argument against this?
6. A positive charge is brought very near to an uncharged insulated conductor. The conductor is grounded while the charge is kept near. Is the conductor charged positively, negatively, or not at all if (a) the charge is taken away and then the ground connection is removed and (b) the ground connection is removed and then the charge is taken away?
7. A charged insulator can be discharged by passing it just above a flame. Explain how.
8. If you rub a coin briskly between your fingers, it will not seem to become charged by friction. Why?
9. If you walk briskly across a carpet, you often experience a spark on touching a door knob. (a) What causes this? (b) How might it be prevented?
10. Why do electrostatic experiments not work well on humid days?
11. Why is it recommended that you touch the metal frame of your personal computer before installing any internal accessories?
12. An insulated rod is said to carry an electric charge. How could you verify this and determine the sign of the charge?
13. If a charged glass rod is held near one end of an insulated uncharged metal rod as in Fig. 25-14, electrons are drawn to one end, as shown. Why does the flow of electrons cease? After all, there is an almost inexhaustible supply of them in the metal rod.

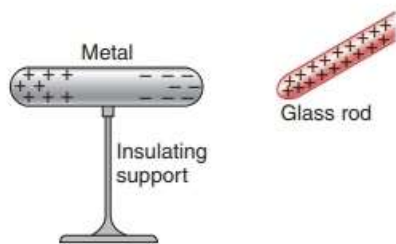


FIGURE 25-14. Questions 13 and 14

14. In Fig. 25-14, does any resultant electric force act on the metal rod? Explain.
15. A person standing on an insulating stool touches a charged, insulated conductor. Is the conductor discharged completely?
16. (a) A positively charged glass rod attracts a suspended object.

Can we conclude that the object is negatively charged? (b) A positively charged glass rod repels a suspended object. Can we conclude that the object is positively charged?

17. Explain what is meant by the statement that electrostatic forces obey the principle of superposition.
18. Is the electric force that one charge exerts on another changed if other charges are brought nearby?
19. A solution of copper sulfate is a conductor. What particles serve as the charge carriers in this case?
20. If the electrons in a metal such as copper are free to move about, they must often find themselves headed toward the metal surface. Why do they not keep on going and leave the metal?
21. Would it have made any important difference if Benjamin Franklin had chosen, in effect, to call electrons positive and protons negative?
22. Coulomb's law predicts that the force exerted by one point charge on another is proportional to the product of the two charges. How might you go about testing this aspect of the law in the laboratory?
23. Explain how an atomic nucleus can be stable if it is composed of particles that are either neutral (neutrons) or carry like charges (protons).
24. An electron (charge = $-e$) circulates around a helium nucleus (charge = $+2e$) in a helium atom. Which particle exerts the larger force on the other?
25. The charge of a particle is a true characteristic of the particle, independent of its state of motion. Explain how you can test this statement by making a vigorous experimental check of whether the hydrogen atom is truly electrically neutral.
26. Suppose the charge in Fig. 25-11 were not distributed uniformly along the length of the rod but instead were concentrated at its center and tapered off at the same rate toward either end. Will the force now have a z component? If this rod had the same total charge q as the uniformly charged rod, how will the magnitude of F_y compare with Eq. 25-15? Repeat both questions if the charge is distributed along the rod so that there is a deficiency near the center and the charge density increases at the same rate toward either end.
27. Earnshaw's theorem says that no particle can be in stable equilibrium under the action of electrostatic forces alone. Consider, however, point *P* at the center of a square of four equal positive charges, as in Fig. 25-15. If you put a positive test charge there it might seem to be in stable equilibrium. Every one of the four external charges pushes it toward *P*, yet Earnshaw's theorem holds. Can you explain how?

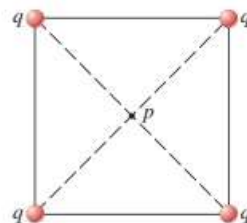


FIGURE 25-15. Question 27.

28. The quantum of charge is 1.60×10^{-19} C. Is there a corresponding quantum of mass?
29. What does it mean to say that a physical quantity is (a) quantized or (b) conserved? Give some examples.
30. In Sample Problem 25-3 we show that the electrical force is about 10^{39} times stronger than the gravitational force. Can

you conclude from this that a galaxy, a star, or a planet must be essentially neutral electrically?

31. How do we know that electrostatic forces are not the cause of gravitational attraction—between the Earth and Moon, for example?

EXERCISES

25-1 Electromagnetism: A Preview

25-2 Electric Charge

1. In the return stroke of a typical lightning bolt (see Fig. 25-16), a current of 2.5×10^4 C/s flows for 20 μ s. How much charge is transferred in this event?



FIGURE 25-16. Exercise 1.

25-3 Conductors and Insulators

25-4 Coulomb's Law

2. What must be the distance between point charge $q_1 = 26.3 \mu\text{C}$ and point charge $q_2 = -47.1 \mu\text{C}$ for the attractive electrical force between them to have a magnitude of 5.66 N?
3. A point charge of $+3.12 \times 10^{-6}$ C is 12.3 cm distant from a second point charge of -1.48×10^{-6} C. Calculate the magnitude of the force on each charge.
4. Two equally charged particles, held 3.20 mm apart, are released from rest. The initial acceleration of the first particle is observed to be 7.22 m/s^2 and that of the second to be 9.16 m/s^2 . The mass of the first particle is 6.31×10^{-7} kg. Find (a) the mass of the second particle and (b) the magnitude of the common charge.
5. Figure 25-17a shows two charges, q_1 and q_2 , held a fixed distance d apart. (a) Find the strength of the electric force that acts on q_1 . Assume that $q_1 = q_2 = 21.3 \mu\text{C}$ and $d = 1.52$ m. (b) A third charge $q_3 = 21.3 \mu\text{C}$ is brought in and placed as

shown in Fig. 25-17b. Find the strength of the electric force on q_1 now.

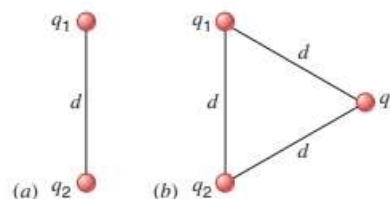


FIGURE 25-17. Exercise 5.

6. Two identical conducting spheres, 1 and 2, carry equal amounts of charge and are fixed a distance apart that is large compared with their diameters. The spheres repel each other with an electrical force of 88 mN. Suppose now that a third identical sphere 3, having an insulating handle and initially uncharged, is touched first to sphere 1, then to sphere 2, and finally removed. Find the force between spheres 1 and 2 now. See Fig. 25-18.

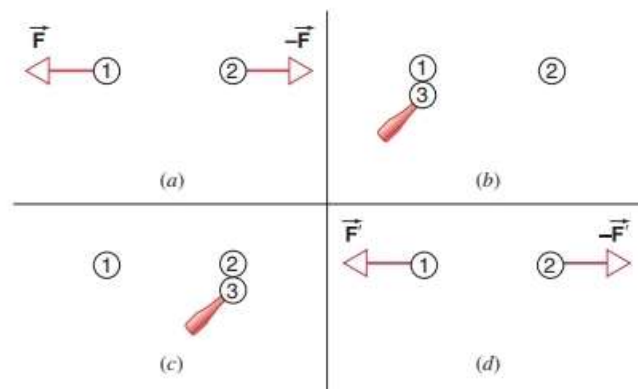


FIGURE 25-18. Exercise 6.

7. Three charged particles lie on a straight line and are separated by a distance d as shown in Fig. 25-19. Charges q_1 and q_2 are held fixed. Charge q_3 , which is free to move, is found to be in equilibrium under the action of the electric forces. Find q_1 in terms of q_2 .



FIGURE 25-19. Exercise 7.

8. In Fig. 25-20, find (a) the horizontal components and (b) the vertical components of the resultant electric force on the charge in the lower left corner of the square. Assume that $q = 1.13 \mu\text{C}$ and $a = 15.2 \text{ cm}$. The charges are at rest.

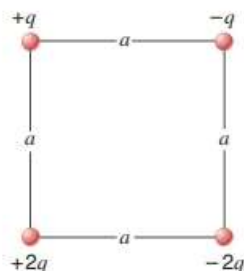


FIGURE 25-20. Exercise 8.

9. Two positive charges, each of $4.18 \mu\text{C}$, and a negative charge, $-6.36 \mu\text{C}$, are fixed at the vertices of an equilateral triangle of side 13.0 cm . Find the electrical force on the negative charge.
10. Each of two small spheres is charged positively, the total charge being $52.6 \mu\text{C}$. Each sphere is repelled from the other with a force of 1.19 N when the spheres are 1.94 m apart. Calculate the charge on each sphere.
11. Two fixed charges, $+1.07 \mu\text{C}$ and $-3.28 \mu\text{C}$, are 61.8 cm apart. Where may a third charge be located so that no net force acts on it?
12. Three small balls, each of mass 13.3 g , are suspended separately from a common point by silk threads, each 1.17 m long. The balls are identically charged and hang at the corners of an equilateral triangle 15.3 cm on a side. Find the charge on each ball.
13. A cube of edge a carries a point charge q at each corner. Show that the resultant electric force on any one of the charges is given by

$$F = \frac{0.262q^2}{\epsilon_0 a^2},$$

directed along the body diagonal away from the cube.

25-5 Continuous Charge Distributions

14. Equation 25-15 was derived assuming the charge q_0 was located on the positive y axis. (a) Does Eq. 25-15 remain valid if the charge is located at negative y ? Explain. (b) Write an equation similar to Eq. 25-15 if the point charge q_0 is instead located a distance x from the rod on the positive or negative x axis. (c) Write an equation in vector component form for the force when q_0 is located a distance d from the rod on the 45° line that bisects the positive x and y axes. (d) Write an equation in vector component form that gives the force when q_0 is located at an arbitrary point x, y anywhere in the xy plane. Check that the components have the correct signs when the point x, y is located in each of the four quadrants.
15. (a) Starting with Eq. 25-16, write an equation in vector form that gives the force when q_0 is located either on the positive or negative z axis of the ring of charge. (b) Do the same for the disk of charge using Eq. 25-17.
16. Find the force on a positive point charge q located a distance x from the end of a rod of length L with uniformly distributed positive charge Q . (See Fig. 25-21.)



FIGURE 25-21. Exercise 16.

17. Consider the rod and charge q_0 in Fig. 25-11. Where can you place a second point charge q (equal to the charge on the rod) so that q_0 is in equilibrium (ignore gravity)? Solve this problem assuming that (a) q is positive and (b) q is negative.
18. Show that the equilibrium of q_0 in Exercise 17 is unstable. (Hint: This problem can be solved by symmetry arguments, and actually requires very little math!)
19. Assume that the rod in Fig. 25-11 has a uniform positive charge density λ on the top half of the rod and a uniform charge density $-\lambda$ on the bottom half of the rod. Find the net force on the point charge q_0 .
20. Four charged rods form the sides of a square in the horizontal (xy) plane. Each rod has a length $L = 25.0 \text{ cm}$ and each carries a uniformly distributed positive charge Q . A small sphere, which can be considered to be a point charge of mass $3.46 \times 10^{-4} \text{ g}$ and charge $q = +2.45 \times 10^{-12} \text{ C}$, is in equilibrium a distance $z = 21.4 \text{ cm}$ above the center of the square. Find the value of Q .

25-6 Conservation of Charge

21. Identify the element X in the following nuclear reactions:
 (a) $^1\text{H} + ^9\text{Be} \rightarrow \text{X} + \text{n}$;
 (b) $^{12}\text{C} + ^1\text{H} \rightarrow \text{X}$;
 (c) $^{15}\text{N} + ^1\text{H} \rightarrow ^4\text{He} + \text{X}$.
 (Hint: See Appendix E.)
22. In the radioactive decay of ^{238}U ($^{238}\text{U} \rightarrow ^4\text{He} + ^{234}\text{Th}$), the center of the emerging ^4He particle is, at a certain instant, $12 \times 10^{-15} \text{ m}$ from the center of the residual ^{234}Th nucleus. At this instant, (a) what is the force on the ^4He particle, and (b) what is its acceleration?
23. In a crystal of salt, an atom of sodium transfers one of its electrons to a neighboring atom of chlorine, forming an ionic bond. The resulting positive sodium ion and negative chlorine ion attract each other by the electrostatic force. Calculate the force of attraction if the ions are 282 pm apart.
24. The electrostatic force between two identical ions that are separated by a distance of $5.0 \times 10^{-10} \text{ m}$ is $3.7 \times 10^{-9} \text{ N}$. (a) Find the charge on each ion. (b) How many electrons are missing from each ion?
25. A neutron is thought to be composed of one "up" quark of charge $+\frac{2}{3}e$ and two "down" quarks each having charge $-\frac{1}{3}e$. If the down quarks are $2.6 \times 10^{-15} \text{ m}$ apart inside the neutron, what is the repulsive electrical force between them?
26. (a) How many electrons would have to be removed from a penny to leave it with a charge of $+1.15 \times 10^{-7} \text{ C}$? (b) To what fraction of the electrons in the penny does this correspond? See Sample Problem 25-1.
27. An electron is in a vacuum near the surface of the Earth. Where should a second electron be placed so that the net

force on the first electron, due to the other electron and to gravity, is zero?

28. Find the total charge in coulombs of 75.0 kg of electrons.
29. Calculate the number of coulombs of positive charge in a glass of water. Assume the volume of the water to be 250 cm^3 .
30. Two physics students (Mary at 52.0 kg and John at 90.7 kg) are 28.0 m apart. Let each have a 0.01% imbalance in their amounts of positive and negative charge, one student being

positive and the other negative. Estimate the electrostatic force of attraction between them. (Hint: Replace the students by spheres of water and use the result of Exercise 29.)

31. (a) What equal amounts of positive charge would have to be placed on the Earth and on the Moon to neutralize their gravitational attraction? Do you need to know the Moon's distance to solve this problem? Why or why not? (b) How many metric tons of hydrogen would be needed to provide the positive charge calculated in part (a)? The molar mass of hydrogen is 1.008 g/mol .

PROBLEMS

1. Two identical conducting spheres, having charges of opposite sign, attract each other with a force of 0.108 N when separated by 50.0 cm . The spheres are suddenly connected by a thin conducting wire, which is then removed, and thereafter the spheres repel each other with a force of 0.0360 N . What were the initial charges on the spheres?
2. A charge Q is fixed at each of two opposite corners of a square. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is zero, how are Q and q related? (b) Could q be chosen to make the resultant electrical force on every charge zero? Explain your answer.
3. Two free point charges $+q$ and $+4q$ are a distance L apart. A third charge is placed so that the entire system is in equilibrium. (a) Find the sign, magnitude, and location of the third charge. (b) Show that the equilibrium is unstable.
4. Two similar tiny balls of mass m are hung from silk threads of length L and carry equal charges q as in Fig. 25-22. Assume that θ is so small that $\tan \theta$ can be replaced by its approximate equal, $\sin \theta$. (a) To this approximation show that, for equilibrium,

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3},$$

where x is the separation between the balls. (b) If $L = 122 \text{ cm}$, $m = 11.2 \text{ g}$, and $x = 4.70 \text{ cm}$, what is the value of q ?

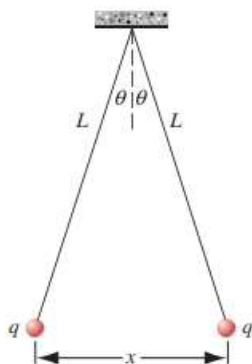


FIGURE 25-22. Problems 4, 5, and 6.

5. If the balls of Fig. 25-22 are conducting, (a) what happens to them after one is discharged? Explain your answer. (b) Find the new equilibrium separation.
6. Assume that each ball in Problem 4 is losing charge at the rate of 1.20 nC/s . At what instantaneous relative speed ($= dx/dt$) do the balls approach each other initially?
7. A certain charge Q is to be divided into two parts, $Q - q$ and q . What is the relation of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?
8. Two positive charges $+Q$ are held fixed a distance d apart. A particle of negative charge $-q$ and mass m is placed midway between them, then is given a small displacement perpendicular to the line joining them and released. Show that the particle describes simple harmonic motion of period $(\epsilon_0 m \pi^3 d^3 / qQ)^{1/2}$.
9. Calculate the period of oscillation for a particle of positive charge $+q$ displaced from the midpoint and along the line joining the two charges $+Q$ in Problem 8.
10. In the compound CsCl (cesium chloride), the Cs atoms are situated at the corners of a cube with a Cl atom at the cube's center. The edge length of the cube is 0.40 nm ; see Fig. 25-23. The Cs atoms are each deficient in one electron and the Cl atom carries one excess electron. (a) What is the strength of the net electric force on the Cl atom resulting from the eight Cs atoms shown? (b) Suppose that the Cs atom marked with an arrow is missing (crystal defect). What now is the net electric force on the Cl atom resulting from the seven remaining Cs atoms?

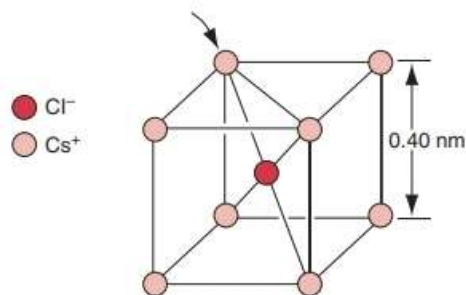


FIGURE 25-23. Problem 10.

11. Two equal positive point charges q are held a fixed distance $2a$ apart. A point test charge is located in a plane that is normal to the line joining these charges and midway between them. Find the radius R of the circle in this plane for which the force on the test particle has a maximum value. See Fig. 25-24.

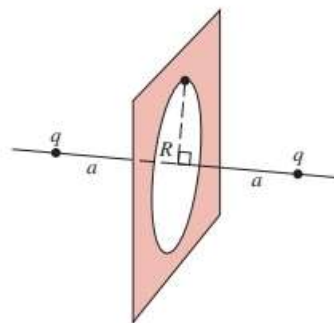


FIGURE 25-24. Problem 11.

COMPUTER PROBLEMS

1. Calculate the force of attraction between two rings with uniformly distributed charges $+q$ and $-q$. The axis of the rings is the x axis, each has a radius R , and the rings are separated by a distance of $2R$. Your final answer should be of the form $F = C_r q^2 / 4\pi\epsilon_0 R^2$, where C_r is a dimensionless constant that you will find.
2. Repeat Computer Problem 1 for the case of two disks with uniformly distributed charges $+q$ and $-q$. Your final answer will still be of the form $F = C_d q^2 / 4\pi\epsilon_0 R^2$, where C_d is a dimensionless constant that you will find; C_d for the disk is *different* from C_r for the ring.
3. Calculate the force of attraction between two solid spheres with uniformly distributed charges $+q$ and $-q$. The spheres

are centered on the x axis, each has a radius R , and the *centers* of the spheres are separated by a distance of $d > 2R$. Your final answer should be of the form $F = C_s q^2 / 4\pi\epsilon_0 d^2$, where C_s is a dimensionless constant that you will find.

4. A uniform ring of charge Q has a radius $R = 1.00$ cm. An electron is constrained to move in the plane of the ring. (a) Assuming that $Q = -100 \mu\text{C}$, find the speed of an electron that would move in a circular orbit of radius $r = 0.50$ cm, concentric with the ring. (b) Assuming that $Q = +100 \mu\text{C}$, find the speed of an electron that would move in a circular orbit of radius $r = 1.50$ cm, concentric with the ring. (c) Show, by numerically integrating the motion, that neither orbit is stable.